



PAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY
FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	LEVEL: 6
COURSE CODE: RAA602S	COURSE NAME: REGRESSION ANALYSIS AND ANALYSIS OF VARIANCE
SESSION: NOVEMBER 2019	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr. D. NTIRAMPEBA Mr. R. MUMBUU
MODERATOR:	Dr. C. R. KIKAWA
INSTRUCTIONS	
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil. Marks will not be awarded for answers obtained without showing the necessary steps leading to them.	

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

ATTACHMENTS

1. Statistical tables (Z, T, Chi-square, and F tables)

THIS QUESTION PAPER CONSISTS OF 5 PAGES (Excluding this front page and Tables)

QUESTION 1 [20 MARKS]

The grades of a class of 9 students on a midterm report (x) and on the final examination (y) are as follows:

x	77	50	71	72	81	94	96	99	67
y	82	66	78	34	47	85	99	99	68

- 1.1. Fit the model $Y = \beta_0 + \beta_1 x + \varepsilon$ to these data.
[5]
- 1.2. Obtain a point estimate of:
 - 1.2.1 the variance of ε , the random deviation in the model equation
[2]
 - 1.2.2 the variance of $\hat{\beta}_0$ [2]
 - 1.2.3 the variance of $\hat{\beta}_1$ [2]
 - 1.3.4 The $cov(\hat{\beta}_0, \hat{\beta}_1)$ [2]
- 1.4. Use a 5% significance of level to test for the significance of the slope of the model
[7]

QUESTION 2 [33 MARKS]

2.1

The district sales manager for a major automobile manufacturer is studying car sales. Specifically, he would like to determine what factors affect the number of cars sold at a dealership. To investigate, he randomly selects 12 dealers. From the dealers, he obtains the number of cars sold last month, the minutes of radio advertising purchased last month, the number of full-time salespeople employed in the dealership, and whether the dealer is in the city. For the variable "whether the dealer is in the city", assume the value of one (1) if the dealer is located in the city, otherwise zero (0). In short, the district sales manager wishes to fit a model of this type

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

The information obtained is as follows:

Advertising	Sales force	City	Cars sold last month
18	10	yes	127
15	15	no	138
22	14	yes	159
23	12	yes	144
17	12	no	139
16	12	yes	128
25	14	yes	161
26	17	yes	180
15	7	no	102
24	16	yes	163
18	10	no	106
25	11	yes	149

The given data were analysed using R-software and below are some of the computer readouts.

$$(X^t X)^{-1} = \begin{bmatrix} 3.41621577 & -0.138861098 & -0.082702085 & 0.786603916 \\ -0.13886110 & 0.012332319 & -0.005594463 & -0.062947914 \\ -0.08270208 & -0.005594463 & 0.015782919 & -0.001245483 \\ 0.78660392 & -0.062947914 & -0.001245483 & 0.763358311 \end{bmatrix}$$

$$\text{and } X^t Y = \begin{bmatrix} 1696 \\ 35357 \\ 21833 \\ 1211 \end{bmatrix}$$

2.1.1 Estimate the multiple linear regression model for the given data. [6]

2.1.2 Construct the 95% confidence interval for regression coefficient of the variable X_3 and use it to deduce whether or not X_3 contributes significant information for prediction of Y . [15]

2.2

Efficient design of certain types of municipal waste incinerators requires that information about energy content of the waste be available. The authors of the article “Modeling the Energy Content of Municipal Solid Waste Using Multiple Regression Analysis” (*J. of the Air and Waste Mgmt. Assoc.*, 1996: 650–656) kindly provided us with data (although not shown here) on Y = energy content (Kcal/kg), the three physical composition variables X_1 = % plastics by weight, X_2 = % paper by weight, and X_3 = % garbage by weight, and the proximate analysis variable X_4 = % moisture (water) by weight for waste specimens obtained from a certain region.

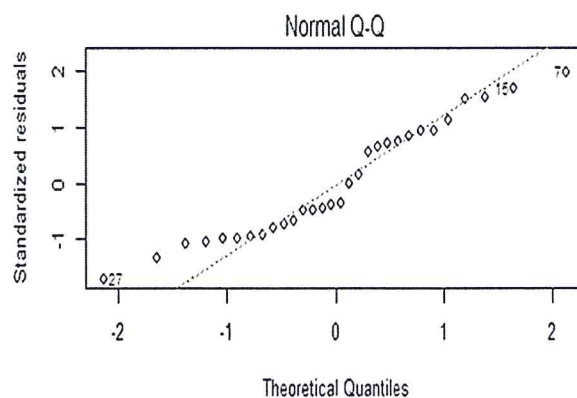
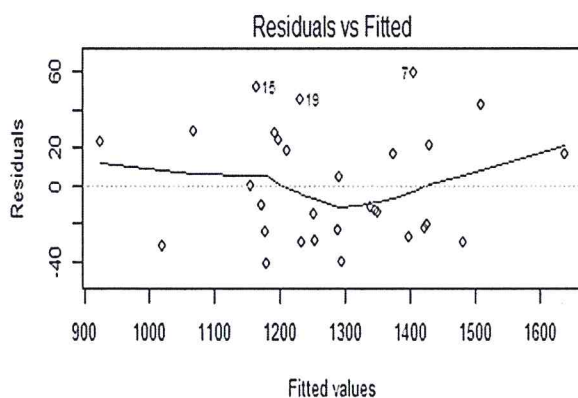
Using R-software to analyse the data and ultimately fit a multiple regression model with the four aforementioned variables as predictors of energy content resulted in the following output:

Model summary

Predictor	Coef	StDev	T	P
Constant	2244.9	177.9	12.62	0.000
plastics	28.925	2.824	-	0.000
paper	7.644	2.314	3.30	0.003
garbage	4.297	1.916	2.24	0.034
water	-37.354	1.834	-20.36	0.000

Analysis of	Variance				
Source	DF	SS	MS	F	P
Regression	4	664931	166233	167.71	0.000
Error	25	24779	991		
Total	29	689710			

Charts



- 2.2.1 Compute and interpret the coefficient of determination. [3]
- 2.2.2 Compute and interpret the adjusted coefficient of determination. [3]
- 2.2.3 Compute the test statistic (t-value) associated with the variable "Plastics". [2]
- 2.2.4 Using information provided, can we say that
 - (a) the assumption of normality is met? Motivate your answer? [2]
 - (b) the assumption of constant variance of errors is constant? Motivate your answer. [2]

QUESTION 3 [33 MARKS]

3.1

Briefly explain the following terminologies as they are applied to Regression Analysis and Analysis of Variance.

- 3.1.1 Experimental design [2]
- 3.1.2 Experimental units [2]
- 3.1.3 Nuisance factor [2]

3.2

The study of Loss of Nitrogen Through Sweat by Preadolescent Boys Consuming Three Levels of Dietary Protein was conducted by the Department of Human Nutrition and Foods at Virginia Tech to determine perspiration nitrogen loss at various dietary protein levels.

Twelve preadolescent boys ranging in age from 7 years, 8 months to 9 years, 8 months, all judged to be clinically healthy, were used in the experiment. Each boy was subjected to one of three controlled diets in which 29, 54, or 84 grams of protein were consumed per day.

The following data represent the body perspiration nitrogen loss, in milligrams, during the last two days of the experimental period: Use $\alpha = 0.05$ level of significance.

29 grams	54 grams	84 grams
190	318	390
266	295	321
270	271	396
	438	399
	402	

- 3.2.1 Write down an appropriate means model for the data. [4]
- 3.2.2 State the null hypothesis and the alternative hypothesis. [2]
- 3.2.3 What is the rejection rule? [1]
- 3.2.4 Compute $SST, SSE, \text{ and } SSTOT$. [6]
- 3.2.5 Construct an ANOVA table. [4]
- 3.2.6 State your decision about the null hypothesis (at 5% significance of level). [1]

- 3.2.7 What is your conclusion (at 5% significance of level)? [1]
- 3.2.8 Estimate the overall mean and the treatment effects. [4]
- 3.2.9 Construct a 95% confidence interval for the difference between the means of nitrogen loss for the first (29 mg) and second diets (54 mg), respectively. [4]

QUESTION 4 [14 MARKS]

The results below are from a study to determine the predictors of fever among children under five years. The dependent variable was “fever” (0=child had no fever in the last two weeks / 1= child had fever in the last two weeks). The seven potential predictor variables are type of place of residence (1= urban/2=rural), sex of the child (1=male/2=female), diarrhea (0= child had no diarrhea in last two weeks/1= child had diarrhea in last two weeks), Vaccination (0= child had no vaccination in last two weeks/1=child had vaccination in last two weeks), Vitamin A (0=child had no vitamin A in last six months/1=child had no vitamin A in last six months), BMI (body mass index) and age (in months).

Covariate	Parameter Estimates	Std. Error	95% Wald C I		Wald Stat
			Lower	Upper	
(Intercept)	0.256	0.6217	-0.963	1.474	0.169
[Place of residence=1]	-0.044	0.2627	-0.559	0.471	0.028
[Place of residence=2 (Ref)]	0
[Sex =1]	-0.168	0.2476	-0.653	0.317	0.46
[Sex =2 (Ref)]	0
[Diarrhae =.00]	-1.308	0.2827	-1.863	-0.754	21.416
[Diarrhae =1.00 (Ref)]	0
[Vaccination =.00]	-0.262	0.6174	-1.472	0.948	0.18
[Vaccination =1.00 (Ref)]	0
[Vitamin A =.00]	-0.623	0.3637	-1.336	0.09	2.934
[Vitamin A =1.00(Ref)]	0
BMI	4.98E-05	0.0002	0	0.001	0.042
Age (in months)	-0.009	0.0076	-0.023	0.006	

- 4.1 What type of analysis was used in this situation? Motivate your answer. [2]
- 4.2 Write down the model for this analysis. [2]
- 4.3 Compute Wald chi-square statistic and use it to test if the variable “Age” is significantly associated with fever (use a 5% significance of level). [5]
- 4.4 Construct the 95 % confidence interval for odds ratio of Diarrhea variable and use your it to infer whether this variable is significantly associated with fever. [5]

END OF QUESTION PAPER

Standard Normal Probabilities

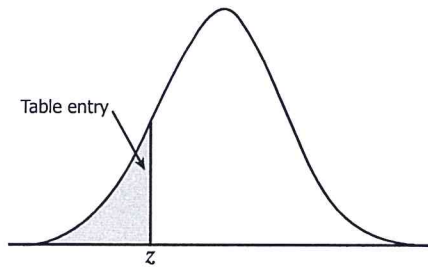
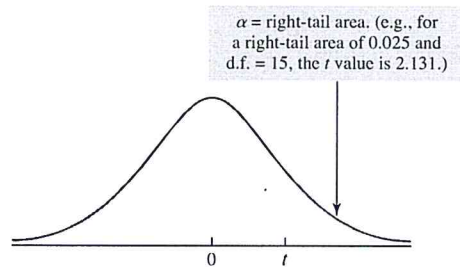


Table entry for z is the area under the standard normal curve to the left of z .

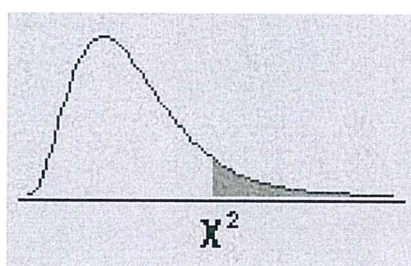
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

The t-Distribution



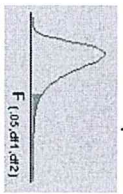
α :	0.10	0.05	0.025	0.01	0.005
d.f. = 1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.435	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690

The Chi-Square Distribution



df/p	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.56200	18.33765	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

F Table for alpha=0.05



df1/df2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	161.4476	199.5	215.7073	224.5832	230.1619	233.996	236.7684	238.8827	240.5433	241.8817	243.906	245.9499	248.0131	249.0518	250.0951	251.1432	252.1957	253.2529	254.3144
2	18.5128	19	19.1643	19.2468	19.2964	19.3295	19.3532	19.371	19.3848	19.3955	19.4125	19.4291	19.4451	19.4541	19.4624	19.4707	19.4791	19.4874	19.4957
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7859	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.572	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.041	5.9988	5.9644	5.9117	5.8758	5.8255	5.7744	5.7459	5.717	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.581	4.5272	4.4957	4.4638	4.4314	4.3985	4.365
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.099	4.06	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.787	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.948	2.8962	2.8536	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.4901	2.448	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.341	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.671	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.8101	2.6987	2.6143	2.548	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.878
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5989	2.514	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.366	2.321	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4424	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.8894	1.8378	1.7831
23	4.2793	3.4221	3.0258	2.7935	2.6381	2.5257	2.4402	2.3728	2.3181	2.2729	2.2018	2.1266	2.0463	2.0035	1.958	1.911	1.861	1.808	1.752
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0272	1.984	1.939	1.892	1.8424	1.7896	1.733
25	4.2417	3.3852	2.9912	2.7587	2.6031	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.711
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.8027	1.7488	1.6906
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.236	2.1901	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.249	2.1802	2.124	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.097	2.0401	1.9926	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.5343	1.4673	1.3933
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0886	2.0184	1.9588	1.9105	1.8337	1.7505	1.6597	1.6084	1.5543	1.4952	1.4329	1.3519	1.2599
Inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5084	1.4591	1.394	1.318	1.2214	1